

THE COUPLING OF THE  $f_1(1285)$  MESON TO THE ISOSCALAR  
AXIAL CURRENT OF THE NUCLEON

M. Kirchbach

Institut für Kernphysik, TH Darmstadt, D-64289 Darmstadt, Germany

and

D.O. Riska

Department of Physics, SF-00014 University of Helsinki, Finland

**Abstract**

The weak decay as well as the strong nucleon coupling constants of the isoscalar axial vector meson  $f_1(1285)$  are estimated within the octet quark model. It is shown that the empirical value for the coupling of  $\bar{s}s$  quarkonium to nucleons can be understood by attributing the  $f_1(1285)$ -nucleon coupling to the  $a_0\pi N$  triangle diagram.

## 1 Introduction

One of the most striking empirical indications for the approximate chiral symmetry of the strong interaction is the existence of the two complete low mass vector meson nonets with opposite parities. An important phenomenological implication of the vector mesons is their role in the electromagnetic structure of the baryons. It was realized early on that the  $\rho$  and  $\omega$  mesons can be used to parametrize the isovector and isoscalar electromagnetic form-factors of the nucleon in terms of (di)pole forms ("vector meson dominance" VMD) [1] –[4]. The VMD idea has also been extended to the parametrization of the axial isovector form factor of the nucleon [3]. This is illustrated e.g. by the phenomenological vector meson Lagrangian proposed in ref.[5], which suggests that the charged weak vector and axial vector nucleon form factors should be expressed as pole terms that contain the masses  $m_\rho$  and  $m_{a_1}$  of the  $\rho$  and  $a_1$  mesons and their respective couplings  $f_\rho$ , and  $f_{a_1}$  to the external  $e\bar{\nu}$  weak current. We here extend the VMD hypothesis to the isoscalar axial form factor of the nucleon along the lines of refs. [6], [7] and present a model for the  $f_1(1285)NN$  coupling constant.

The paper is organized as follows. In section 2 we calculate the weak decay couplings of the  $f_1(1285)$  (subsequently denoted by  $D$ ) and  $f_1(1420)$  (subsequently denoted by  $E$ ) mesons and exploit VMD to derive relations between hadronic and semileptonic couplings. In section 3 we calculate the contribution of the  $D \rightarrow (a_0 + \pi)NN$  triangle diagram to the  $DNN$  vertex and demonstrate its importance for the coupling of the nucleon to an external isoscalar axial vector current. The paper ends with a short summary.

## 2 The couplings of the $f_1(1285)$ meson to neutral lepton and nucleon currents

The neutral weak axial current of the nucleon  $J_{\mu,5}^0$  is determined by the matrix element of the corresponding isos triplet ( $J_{\mu,5}^{(3)}$ ), singlet ( $J_{\mu,5}^{(0)}$ ), and octet ( $J_{\mu,5}^{(8)}$ ) quark currents as

$$\begin{aligned} J_{\mu,5}^0 &= \langle N | \left( -\frac{1}{2} J_{\mu,5}^{(3)} + \frac{1}{2\sqrt{6}} J_{\mu,5}^{(0)} - \frac{1}{2\sqrt{3}} J_{\mu,5}^{(8)} \right) | N \rangle, \\ J_{\mu,5}^{(3,0,8)} &= \bar{\psi} \gamma_\mu \gamma_5 \frac{\lambda^{(3,0,8)}}{2} \psi, \end{aligned} \quad (2.1)$$

where  $\psi$  is the flavour triplet  $(u, d, s)^T$  of up, down and strange quark fields. The VMD hypothesis implies the following field-current identities:

$$\begin{aligned} \langle N | J_{\mu,5}^{(3,0,8)} | N \rangle &= \frac{m_V^2}{2f_V} V_\mu, \\ \langle \bar{N} N | J_\mu^V | V \rangle &= \frac{m_V^2}{2f_V} e_\mu^V, \end{aligned} \quad (2.2)$$

where  $V_\mu = (a_1)_\mu, (D_0)_\mu$ , and  $(E_8)_\mu$ , which denote the fields of the isos triplet, the octet singlet, and the octet isoscalar axial vector mesons, respectively and  $f_V$  are the corresponding decay constants. The VMD hypothesis allows derivation of interesting relations between various hadronic and semileptonic couplings. For example, in describing the  $a_1 + \pi \rightarrow \nu + \bar{\nu}$  annihilation coupling constant  $g_{a_1\pi(\nu\bar{\nu})}$  by means of the chain  $a_1 + \pi \rightarrow D \rightarrow \nu + \bar{\nu}$  (Fig. 1a) the following expression is obtained:

$$g_{a_1\pi(\nu\bar{\nu})} = g_{D a_1 \pi} \frac{m_D^2}{2f_D} \frac{1}{m_D^2 - q^2}. \quad (2.3)$$

Another intriguing relation arises between the pion weak decay coupling constant  $f_\pi$  and the coupling constant  $g_{D a_0(\mu\bar{\nu}_\mu)}$  of the (hypothetical) weak decay reaction  $D \rightarrow a_0 + \mu + \bar{\nu}_\mu$ , which is of interest for the evaluation of short range meson exchange currents in nuclear muon capture reactions of the type displayed in Fig. 1b,

$$g_{D a_0 \pi} \frac{q^2}{q^2 - m_\pi^2} f_\pi e^D(x) \cdot J_5^{(\mu\bar{\nu}_\mu)}(x) \xrightarrow{q^2 \gg m_\pi^2} g_{D a_0(\mu\bar{\nu}_\mu)} e^D(x) \cdot J_5^{(\mu\bar{\nu}_\mu)}(x). \quad (2.4)$$

To obtain this equation we use the Lagrangian density

$$\mathcal{L}_{D a_0 \pi}(x) = g_{D a_0 \pi} e_\lambda^D(x) \partial^\lambda \vec{\pi}(x) \cdot \vec{a}_0(x), \quad (2.5)$$

In eq. (2.4)  $J_5^{(\mu\bar{\nu}_\mu)}(x)$  is the external weak axial lepton current.

In the limit of equal meson masses the nonrelativistic quark model leads to the following relation between these decay constants:

$$\frac{1}{f_{a_1}} : \frac{1}{f_{D_0}} : \frac{1}{f_{E_8}} = 1 : -\frac{1}{\sqrt{6}} : \frac{1}{\sqrt{3}}. \quad (2.6)$$

The numerical value of  $f_{a_1}$  is determined by the second Weinberg sum rule [8] as  $f_{a_1}^2 = f_\rho^2$ , while  $f_\rho$  is given by the KSFR relation as  $f_\rho^2 = m_\rho^2/2f_\pi^2$ , where  $f_\pi$  is the pion decay constant. Thus eq. (2.2) allows expression of the couplings  $f_{D_0}$  and  $f_{E_8}$  in terms of  $f_{a_1}$ . This leads to the following numerical values:

$$f_{D_0} = -\sqrt{6}f_{a_1} = -7.093, \quad (2.7)$$

$$f_{E_8} = \sqrt{3}f_{a_1} = 5.016. \quad (2.8)$$

According to the universal current-current coupling model the meson-nucleon vertices are described by effective Lagrangians of the type

$$\mathcal{L}_{VNN} = f_V V_\eta \bar{\psi} \gamma^\eta \gamma_5 \frac{\lambda^{(3,0,8)}}{2} \psi. \quad (2.9)$$

In the quark model the wave functions of the low lying vector mesons are described as linear combinations of quark-antiquark ( $q\bar{q}$ ) pairs in (three dimensional) flavour space. The physical singlet and isoscalar states of both the vector and axial vector meson nonets are moreover predicted to be mixed by the so called "ideal" mixing angle  $\theta_0 = \tan^{-1} \frac{1}{\sqrt{2}}$  with the consequence that the states  $\bar{s}s$  and  $(\bar{u}u + \bar{d}d)/\sqrt{2}$  decouple completely. This is equivalent to a suppression of  $\bar{s}s \rightarrow (\bar{u}u + \bar{d}d)/\sqrt{2}$  transitions (the Okubo-Zweig-Iizuka rule). Consequently the decay of, say, the  $\phi(\bar{s}s)$  meson into three pions via the chain  $\phi \rightarrow \rho\pi \rightarrow \pi^+\pi^-\pi^0$  is represented in QCD by nonplanar diagrams, which are known to be suppressed relative to the leading planar graphs by  $1/N_c$ . The constants  $f_D$  and  $f_E$  corresponding to the ideally mixed  $D$  and  $E$  mesons are then given as

$$f_D = f_{D_0} \cos\theta_0 - f_{E_8} \sin\theta_0 = -3f_{a_1}, \quad (2.10)$$

$$f_E = f_{D_0} \sin\theta_0 + f_{E_8} \cos\theta_0 = 0. \quad (2.11)$$

In this way the couplings of the isoscalar axial vector mesons to the external lepton current equal (up to the sign) the corresponding couplings of the isoscalar vector mesons for which the following relations have been obtained within the quark model [9],  $f_\omega = 3f_{a_1}$ , and  $f_\phi = 0$ . The experimental values for the physical mixing angle (subsequently denoted by  $\theta$ ), which have been extracted from the empirical meson masses with linear or quadratic mass formulae, disagree however with the ideal one. This difference is comparatively small in the case of the vector meson nonet (about  $5^\circ$ ). Even so, the small observed  $\omega - \phi$  mixing has a non-negligible impact on the weak vector strangeness radius of the nucleon – a result obtained in [10], [11] under the assumption of VMD of the isoscalar weak vector current. The axial vector meson nonet is clearly also non-ideal, as the empirical mixing value for it is  $\theta = \theta_0 + \epsilon = 50^\circ \pm 2.7^\circ \pm 3.6^\circ$  [12].

The wave functions of the isoscalar axial vector mesons that take into account the mixing between the strange and non strange quarkonia are

$$\begin{aligned}
D(1285) \equiv f_1(1285) &= \cos\epsilon \frac{\bar{u}u + \bar{d}d}{\sqrt{2}} - \sin\epsilon \bar{s}s \\
E(1420) \equiv f_1(1420) &= \sin\epsilon \frac{\bar{u}u + \bar{d}d}{\sqrt{2}} + \cos\epsilon \bar{s}s.
\end{aligned} \tag{2.12}$$

The substantial deviation of the D–E mixing from the ideal mixing angle is explained qualitatively in QCD as being due to strong nonperturbative effects in the isoscalar axial channel [13]. In the case of the mixed isoscalar axial vector mesons eqs. (2.9)–(2.10) are replaced by

$$\hat{f}_D = f_D \cos\epsilon - f_E \sin\epsilon = -3f_{a_1} \cos\epsilon, \tag{2.13}$$

$$\hat{f}_E = f_D \sin\epsilon + f_E \cos\epsilon = -3f_{a_1} \sin\epsilon. \tag{2.14}$$

The isoscalar axial vector mesons  $D(1285)$  and  $E(1420)$  are expected to be important as intermediate states in  $N\bar{N}$  annihilation as well as in neutrino scattering processes. In this context the weak decay couplings of the  $f_1(1285)/f_1(1420)$  mesons can be associated with the quantities  $\hat{f}_D/\hat{f}_E$ . The interpretation of the latter as the strong meson–nucleon couplings is, however, not obvious. This follows from the observation that eq. (2.1) in fact reduces to

$$\begin{aligned}
J_{\mu,5}^0 &= -\frac{1}{2} \langle N | J_{\mu,5}^{(3)} | N \rangle + \frac{1}{4} \langle N | \bar{s}\gamma_\mu\gamma_5 s | N \rangle, \\
&= -\frac{g_A}{2} \bar{u}_N \gamma_\mu \gamma_5 \frac{\tau_3}{2} u_N + \frac{1}{2} G_1^s \bar{u}_N \gamma_\mu \gamma_5 u_N,
\end{aligned} \tag{2.15}$$

where  $g_A$  is the weak proton axial coupling constant, and  $G_1^s$  denotes the so called weak isoscalar nucleon coupling ( $G_1^s = -0.13 \pm 0.04$  [14]). This equation suggests that the coupling constants in the  $f_1(1285)NN$  and  $f_1(1420)NN$  vertices (in turn denoted by  $g_{DNN}$ , and  $g_{ENN}$ ) will be determined by the  $\bar{s}s$  components of the corresponding wave functions according to

$$\begin{aligned}
g_{DNN} &= g_{(\bar{s}s)} \sin\epsilon, \\
g_{ENN} &= g_{(\bar{s}s)} \cos\epsilon.
\end{aligned} \tag{2.16}$$

Here  $g_{(\bar{s}s)}$  denotes the coupling of the purely strange quarkonia to the nucleon. By the parity invariance (chiral symmetry) of the strong interaction one can assume that the value  $g_{(\bar{s}s)}$  from the axial vector meson nonet equals the empirical value of  $g_{(\bar{s}s)} = 6.75$ , which is extracted from data on the vector meson nonet [17]. This yields the following relation

$$|g_{DNN}| = \sin\epsilon g_{(\bar{s}s)} = 1.72. \tag{2.17}$$

In the following section we show that by attributing the  $f_1(1285)NN$  coupling to the  $a_0\pi N$  triangle diagram, which is the dominant one loop contribution to the isoscalar weak axial nucleon coupling, the empirical value for the coupling of the  $\bar{s}s$  quarkonium to the nucleon is well understood.

### 3 The $\pi a_0 N$ triangle diagram contribution to $g_{DNN}$

The  $\pi a_0 N$  triangle diagram contribution to the  $D$ -nucleon coupling  $g_{DNN}$  (Fig. 2) and the corresponding form factor can be calculated using the  $D a_0 \pi$  Lagrangian (2.5) and the following Lagrangians for the  $\pi NN$  and  $a_0 NN$  couplings

$$\mathcal{L}_{\pi NN} = \frac{f_{\pi NN}}{m_\pi} \bar{\psi} \gamma_5 \gamma^\mu \partial_\mu \vec{\phi} \cdot \vec{\tau} \psi, \quad (3.1)$$

$$\mathcal{L}_{a_0 NN} = g_{a_0 NN} \bar{\psi} \vec{\phi} \cdot \vec{\tau} \psi. \quad (3.2)$$

Here  $f_{\pi NN}$  and  $g_{a_0 NN}$  in turn denote the pseudovector  $\pi NN$  and the scalar  $a_0 NN$  coupling constants, for which we adopt the values  $f_{\pi NN}^2/4\pi = 0.08$  and  $g_{a_0 NN}^2/4\pi = 1.075$ , respectively. These values are implied by the semiphenomenological Bonn one boson exchange model for the nucleon-nucleon interaction [15]. We define the effective  $D$ -nucleon vertex by the scalar product between the weak isoscalar axial vector current of the nucleon ( $J_{\mu,5}^0$ ) and the  $f_1(1285)$  field (denoted by  $V_D^\mu$ ) as

$$J_5^0 \cdot V_D = iG_1^s \bar{u}(\vec{p}) \gamma_5 \gamma^\mu u(\vec{p}) f_{DNN} e_\mu^D. \quad (3.3)$$

The product  $f_{DNN} G_1^s$  equals now the quantity  $g_{DNN}$  introduced by eq. (2.16) with  $g_{DNN} = g_{DNN}(-m_D^2)$ . To regularize the integral in the triangle diagrams in Fig.2 we introduce the same monopole formfactors at the  $\pi NN$  and  $a_0 NN$  vertices as in the potential model [15]. The two triangle diagrams then give the following contribution to the DNN vertex:

$$g_{DNN}(q^2) = \frac{3}{16\pi^2} g_{\pi NN} g_{D a_0 \pi} g_{a_0 NN} \int_0^1 dx \int_0^1 dy x \log \left\{ \frac{Z(m_\pi, \Lambda_a) Z(m_a, \Lambda_\pi)}{Z(m_\pi, m_a) Z(\Lambda_\pi, \Lambda_a)} \right\}. \quad (3.4)$$

Here  $\Lambda_\pi$  and  $\Lambda_a$  are the mass parameters in the monopole vertex factors, for which we use the values 1.3 GeV and 2.0 GeV respectively, and the function  $Z(m_1, m_2)$  is defined as

$$Z(m_1, m_2) = -q^2 xy^2(x-1) + m_N^2 x^2(1-y)^2 + (1-x)m_1^2 + xym_2^2. \quad (3.5)$$

The DNN coupling constant is obtained by setting  $q^2 = -m_D^2$  in eq. (3.4). Note that as  $m_D > m_\pi + m_{a_0}$  the expression (3.4) has a small imaginary part when the  $D$ -meson is on shell, which we shall ignore. Using for  $g_{D a_0 \pi}$  the value 5.7 from the experimental partial decay width [16], the real part of (3.4) at  $q^2 = -m_D^2$  is then 1.9, which we shall interpret as the value for the DNN coupling constant, i.e.

$$|g_{DNN}| = 1.9. \quad (3.6)$$

The  $q^2$  dependence of  $g_{DNN}$  is shown in Fig.3. The  $q^2$ -dependence corresponds approximately to that of a monopole form with a mass scale parameter  $\Lambda \sim 2$  GeV.

## 4 Results and discussion

It has long been known that the  $\phi$ NN coupling is strongly underpredicted in the quark model as seen from the unexpectedly large value for the  $(\bar{s}s)$ NN coupling extracted from data analyses in [17]. Comparison of our result from eq. (3.6) shows that the numerical value for  $g_{DNN}$  associated with the  $a_0\pi N$  triangle diagram agrees fairly well with the empirical estimate given in eq. (2.17). Consequently, in spite of the small decay width of the  $f_1(1285)$  meson the coupling constant in the  $f_1(1285)NN$  vertex seems to be completely exhausted by the contribution of the  $a_0\pi N$  triangle diagram.

The result that the empirical value for that coupling constant can be explained by the  $\pi a_0$  triangle diagram alone shows that the meson model is able to predict realistic results for reactions that involve the coupling of the  $s\bar{s}$  system to nucleons.

Using the experimental value of  $G_1^s$  from [14] the value of  $f_{DNN}$  is obtained as  $f_{DNN} = -14.61$  which is compatible with the size of  $\hat{f}_D = -3f_{a_1}\cos\epsilon = -17.2$  obtained by means of the KSFR relation. This suggests that the vector meson dominance model may be applied in the case of isoscalar axial vector currents.

### Acknowledgements

M.K. is grateful to the hospitality of the Department of Physics of the University of Helsinki and the Academy of Finland for travel support.

## References

- [1] Y. Nambu, Phys. Rev. **106** (1957) 1366
- [2] W. R. Frazer and J. R. Fulco, Phys. Rev. **117** (1960) 1609
- [3] J. J. Sakurai, Ann. Phys. (N.Y.) **11** (1960) 1
- [4] S. I. Dolinski et al., Phys. Rep. **202** (1991) 101
- [5] V.I Ogievetsky and B. M. Zupnik, Nucl. Phys. **B24** (1970) 612
- [6] M. Hjorth-Jensen, M. Kirchbach, K. Tsushima, and D. O. Riska, Nucl. Phys. **A563** (1993) 525
- [7] M. Kirchbach and H. Arenhövel, Proc. Int. Conf. on "Physics with GeV-Particle beams", Jülich, 22–25 August, 1994, Germany, World Scientific (1995), in press
- [8] S. Weinberg, Phys. Rev. Lett. **18** (1967) 507
- [9] M. M. Nagels, T. A. Rijken and J. J. De Swart, Phys. Rev. **D12** (1975) 744
- [10] R. L. Jaffe, Phys.Lett. **B229** (1989) 275
- [11] T. D. Cohen, H. Forkel, and M. Nielsen, Phys.Lett. **B316** (1993) 1
- [12] T. Bolton et al., Phys. Lett. **B278** (1992) 495
- [13] M. A. Shifman et al., Nucl. Phys. **B147** (1979) 448
- [14] J. Ellis and M. Karliner, Phys. Lett. **B313** (1993) 131
- [15] C. Elster, K. Holinde, and R. Machleidt, Phys.Rept. **149** (1987) 149
- [16] Review of Particle Properties, Phys. Rev. **D50** (1994) 1173
- [17] H. Genz and G. Höhler, Phys. Lett. **61B** (1976) 389

### Figure captions

Fig. 1 (a)  $D$  exchange contribution to  $a_1\pi \rightarrow \nu\bar{\nu}$ , (b)  $\pi$  exchange contribution to  $D \rightarrow a_0\mu\bar{\nu}$ .

Fig. 2 The  $\pi a_0$  triangle diagram contribution to the  $D$ -nucleon coupling.

Fig. 3 The momentum dependence of the  $\pi a_0$  triangle diagram contribution to the  $DNN$  coupling  $g_{DNN}(q^2)$ . The units of  $q^2$  are  $GeV^2$ .

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/nucl-th/9502018v1>

This figure "fig1-2.png" is available in "png" format from:

<http://arXiv.org/ps/nucl-th/9502018v1>

This figure "fig1-3.png" is available in "png" format from:

<http://arXiv.org/ps/nucl-th/9502018v1>